HITS
Heidelberg Institute for Theoretical Studies

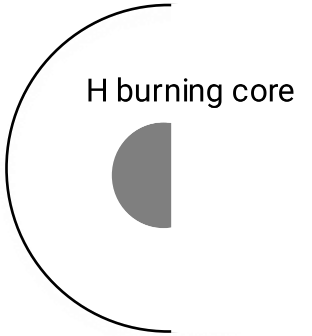
The quest for the ultimate stellar model

Red giants

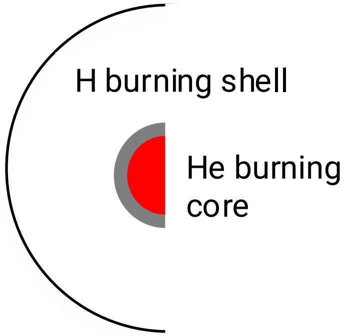
Saskia Hekker

TAs: Susmita Das; Noi Shitrit; Arthur Le Saux; Zhao Guo

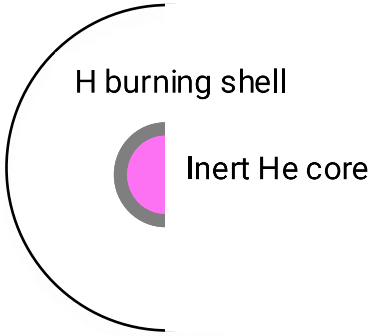
Stellar Structure & Evolution



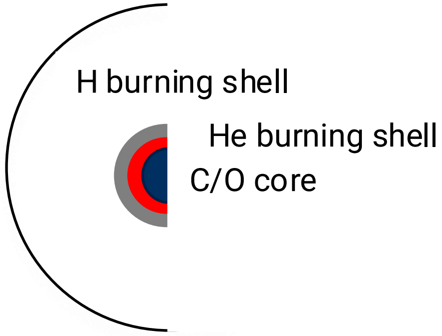
A-B main sequence



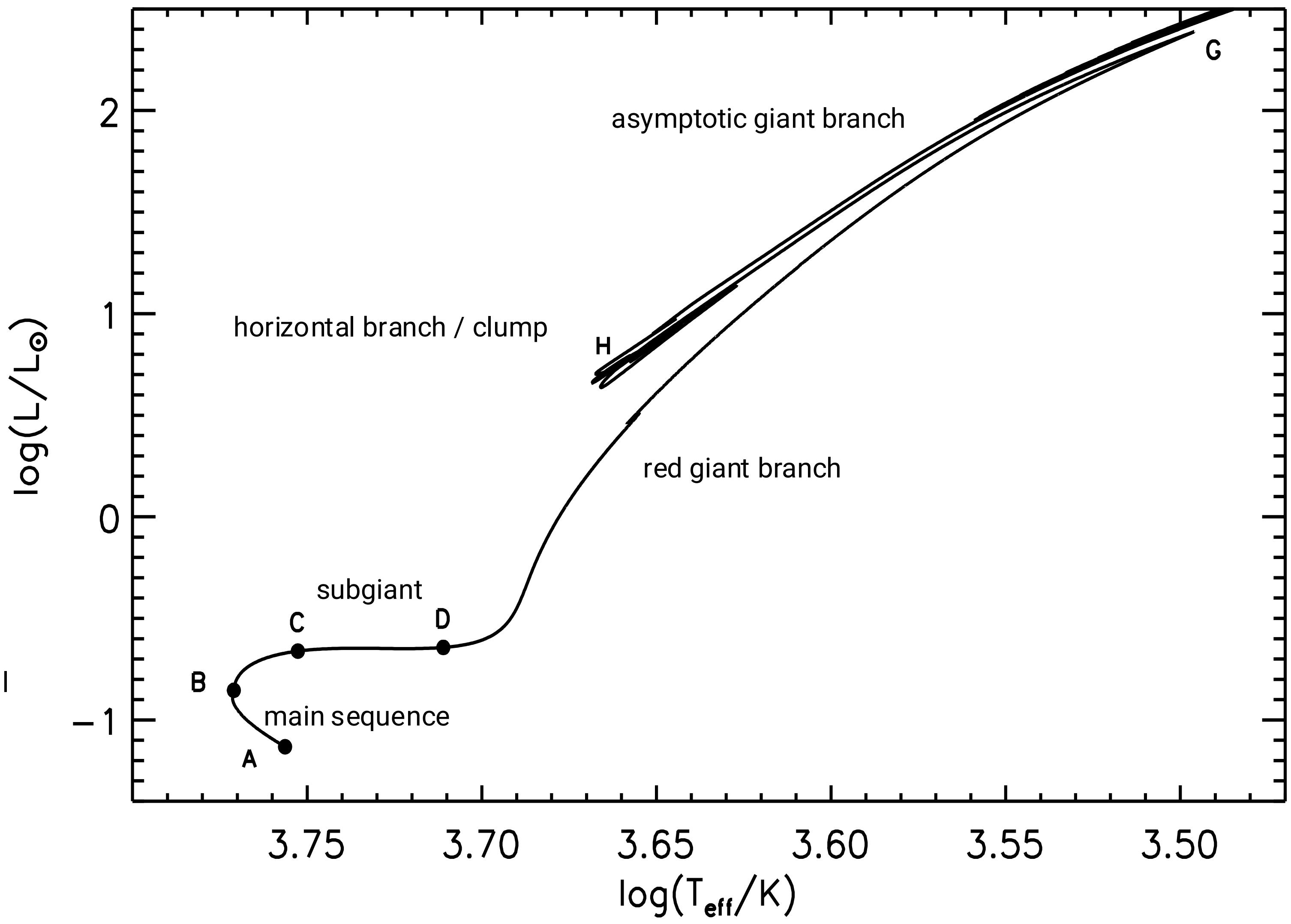
H horizontal branch / clump



B-G subgiant / red giant branch



> H asymptotic giant branch



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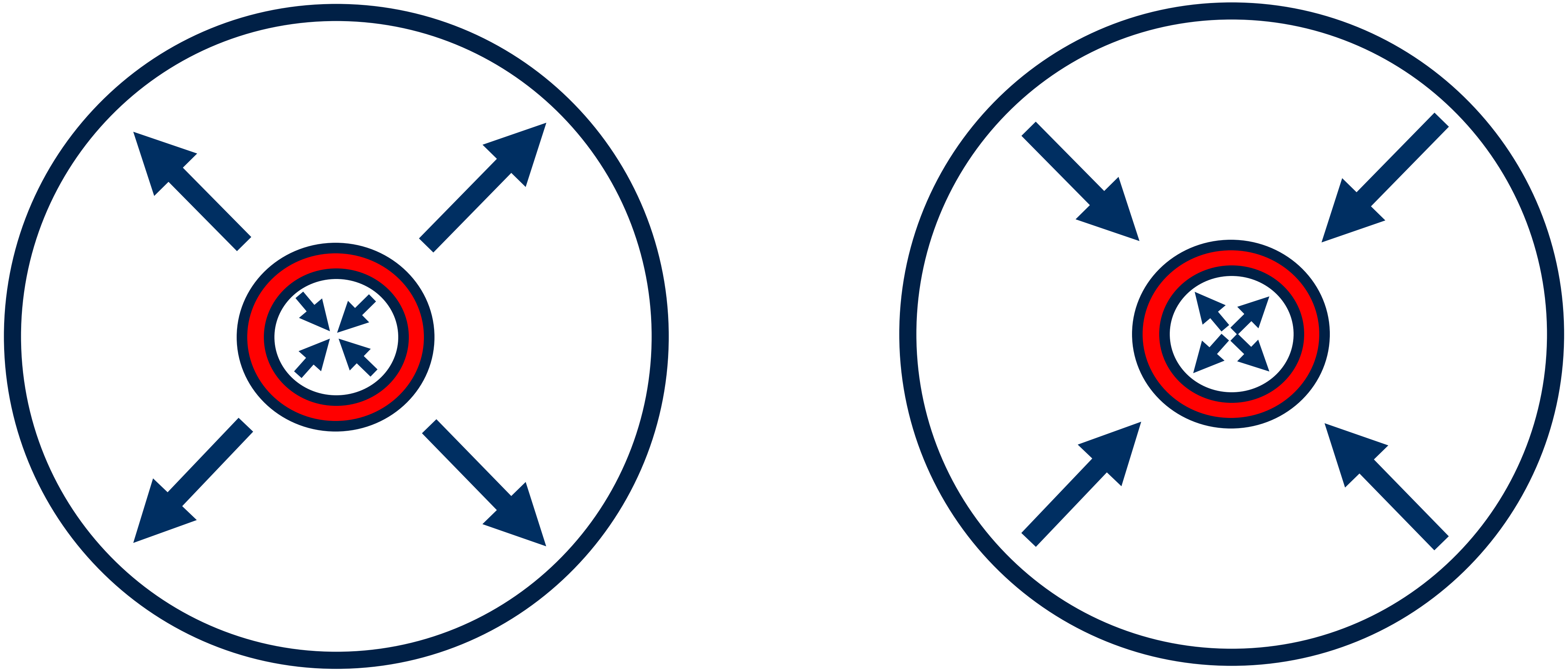
Mirror principle and the red-giant bump: the battle of entropy in low-mass stars

S. Hekker ,[1,2★] G. C. Angelou ,[3] Y. Elsworth[4,2] and S. Basu[5]

Mirror principle



contracting core = expanding envelope

expanding core = contracting envelope

Attributes of mirror

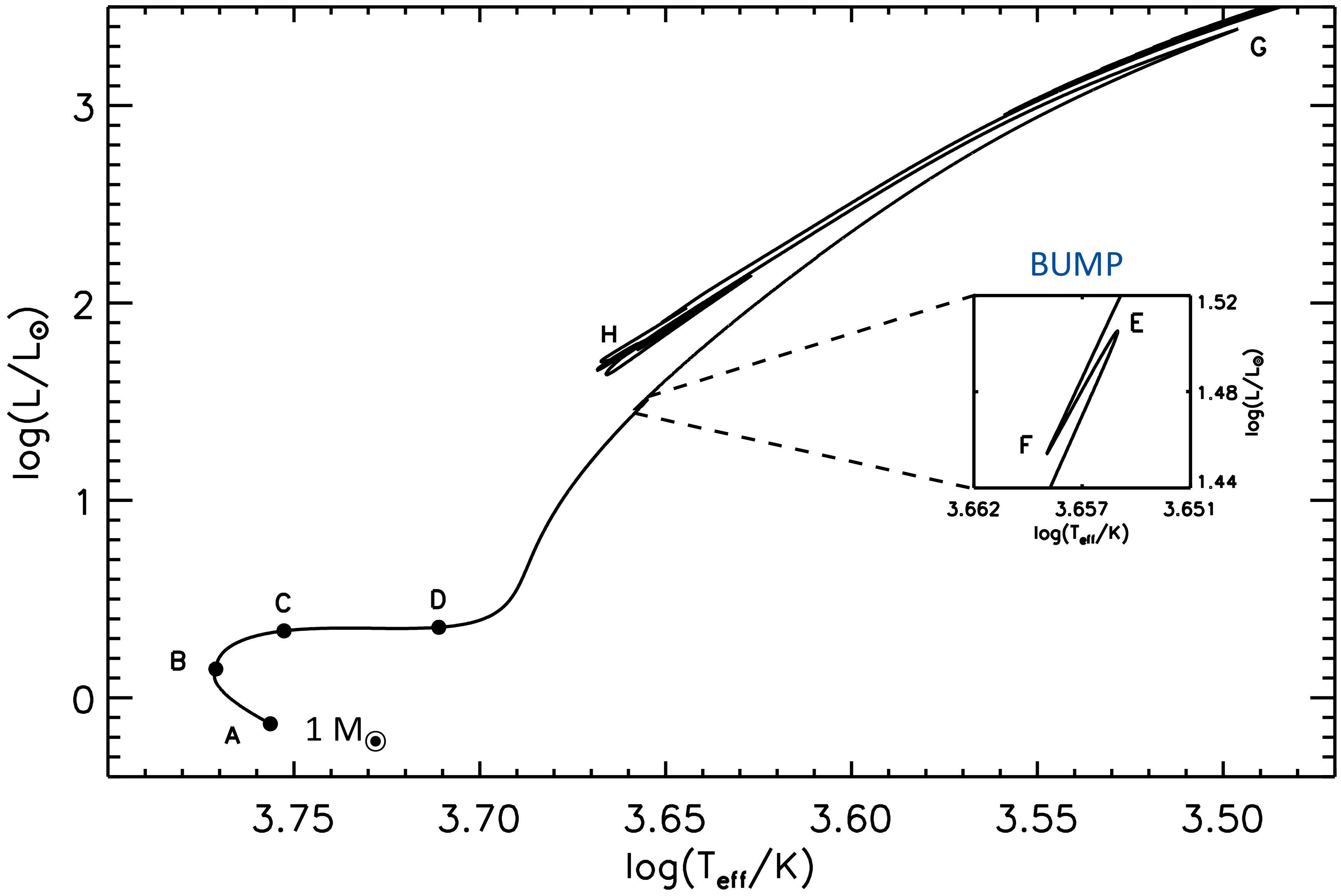
Stationary point: a location below which shells move inwards and above which shells move outwards (or vice versa):

$$\frac{\partial r}{\partial t} = 0$$

Pivot: location where the gravothermal energy generation rate changes sign – the location where shells transition from compressing to expanding

$$\varepsilon_g = -T\frac{\partial s}{\partial t} = 0$$

Red-giant bump



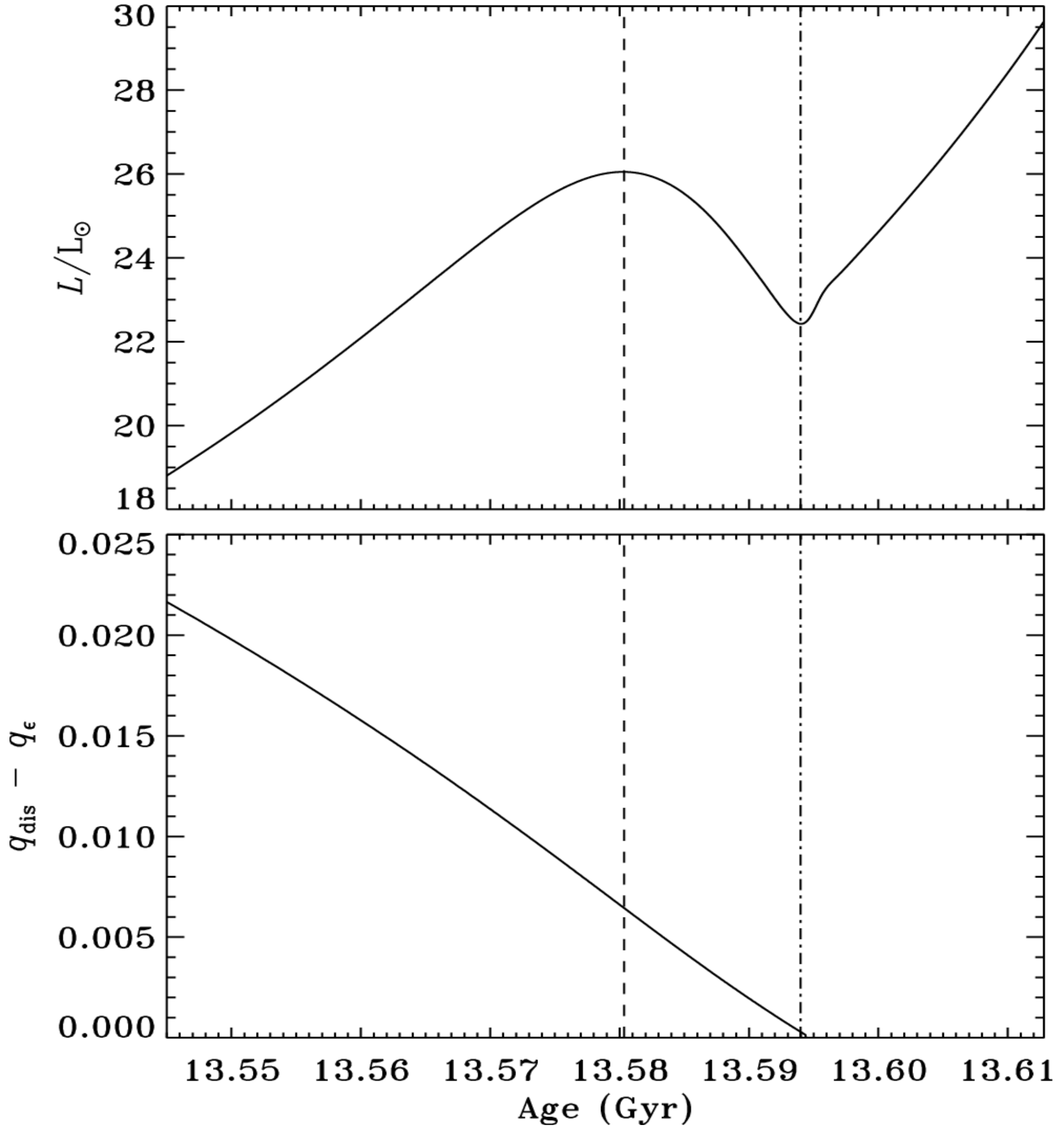
Adapted from Hekker & Christensen-Dalsgaard, 2017

Red-giant bump

The naïve explanation is that the bump appears when the hydrogen shell burns through the mean molecular weight discontinuity left behind by the deepest extent of the convection zone.

At this discontinuity, the amount of hydrogen available for burning increases and consequently there is a re-adjustment of the internal structure.

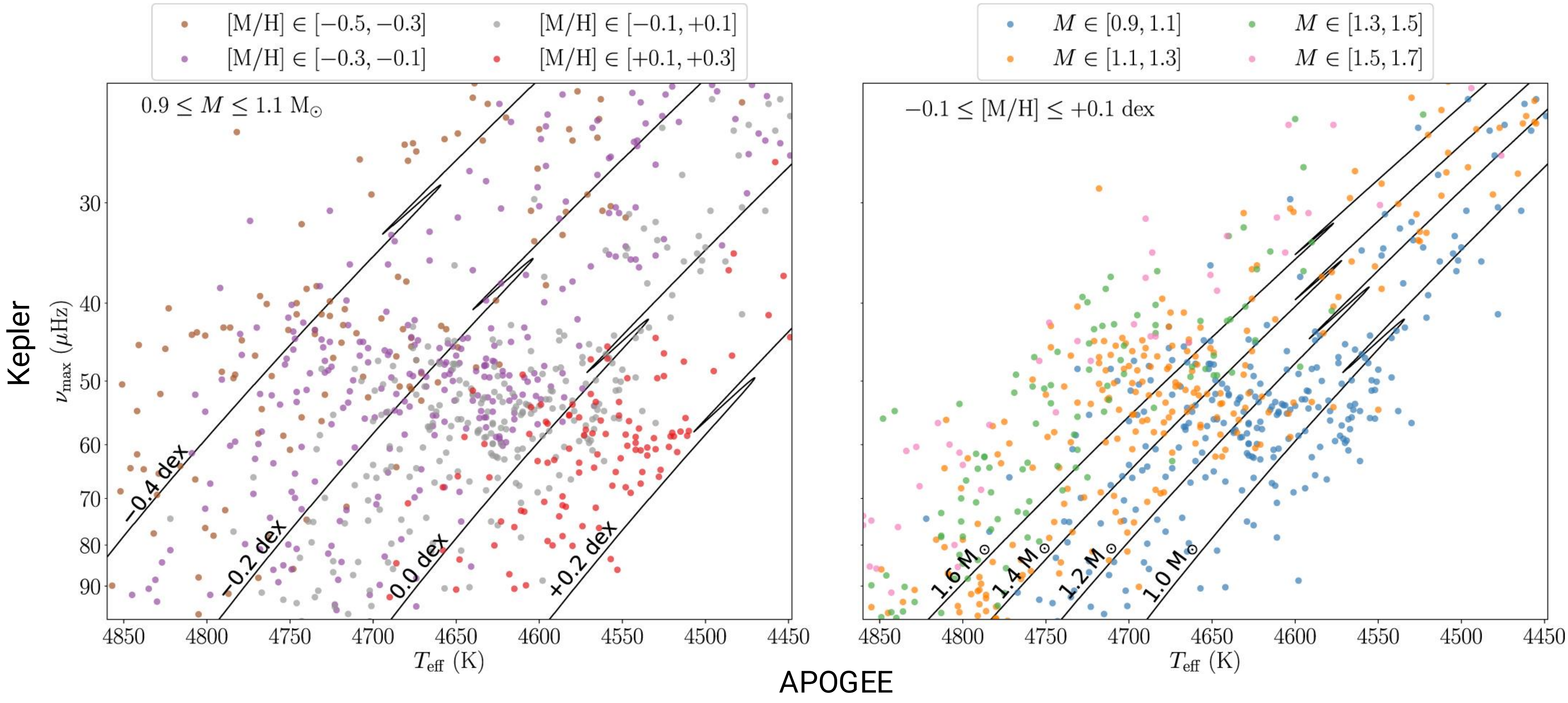
Motivation: H-shell burns through $\boldsymbol{\mu}$-discontinuity at L_{min}



...Even so I find it plausible, as implied by Refsdal & Weigert (1970), that the cause of the luminosity variation in the bump is **predominantly the result of the effect of the composition discontinuity on the hydrostatic equilibrium**, the effective weight of the overlying material being reduced by the decrease in the mean molecular weight above the discontinuity.....

Christensen-Dalsgaard, 2015

Motivation: miss-match bump location



Khan et al. 2018

Motivation: entropy

Sugimoto & Fujimoto 2000: “Why stars become giants?”:

‘An increase of the entropy in the envelope is indispensable for the evolution to a red giant.’

Specific entropy

For a constant composition ideal monoatomic gas specific entropy can be expressed as (Hanson & Kawaler 1994):

$$s = \frac{N_A k_B}{\mu} \ln\left[T^{5/2}/P\right] + c$$

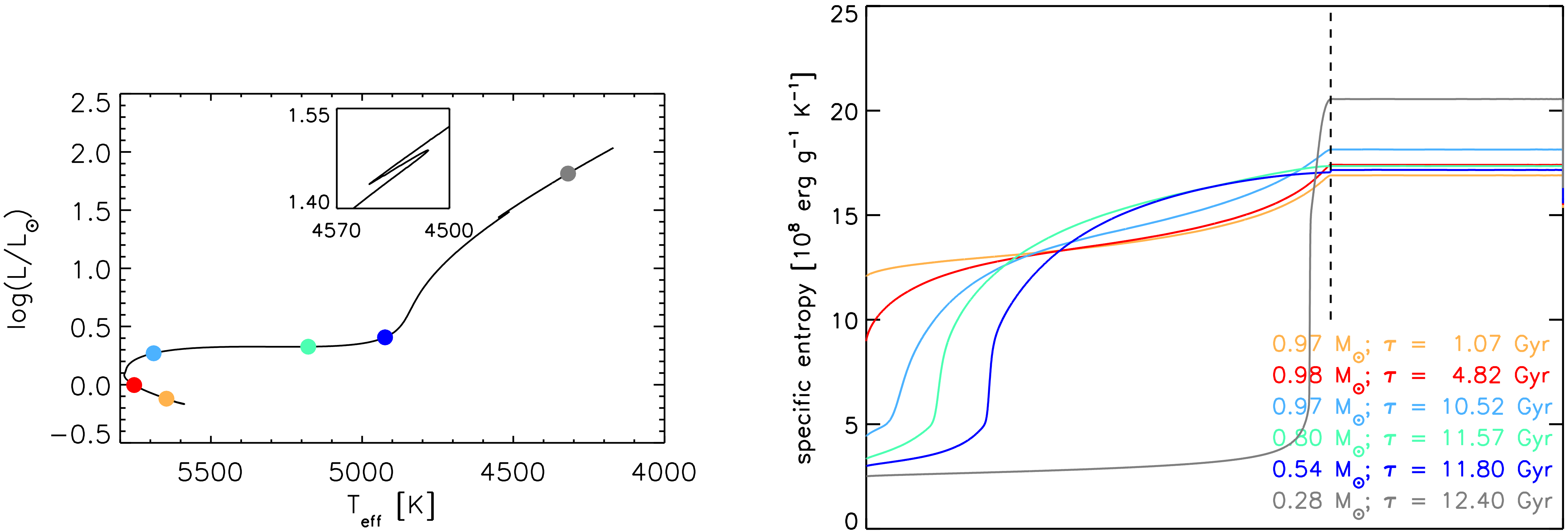
$$\frac{ds}{dr} = c_P(\nabla - \nabla_{ad})\frac{d\ln P}{dr}$$

$$\nabla = \frac{d\ln T}{d\ln P}$$

s = specific entropy
N_A = Avogadro's constant
k_B = Boltzmann's constant
μ = mean molecular weight
T = temperature
P = pressure
c = integration constant

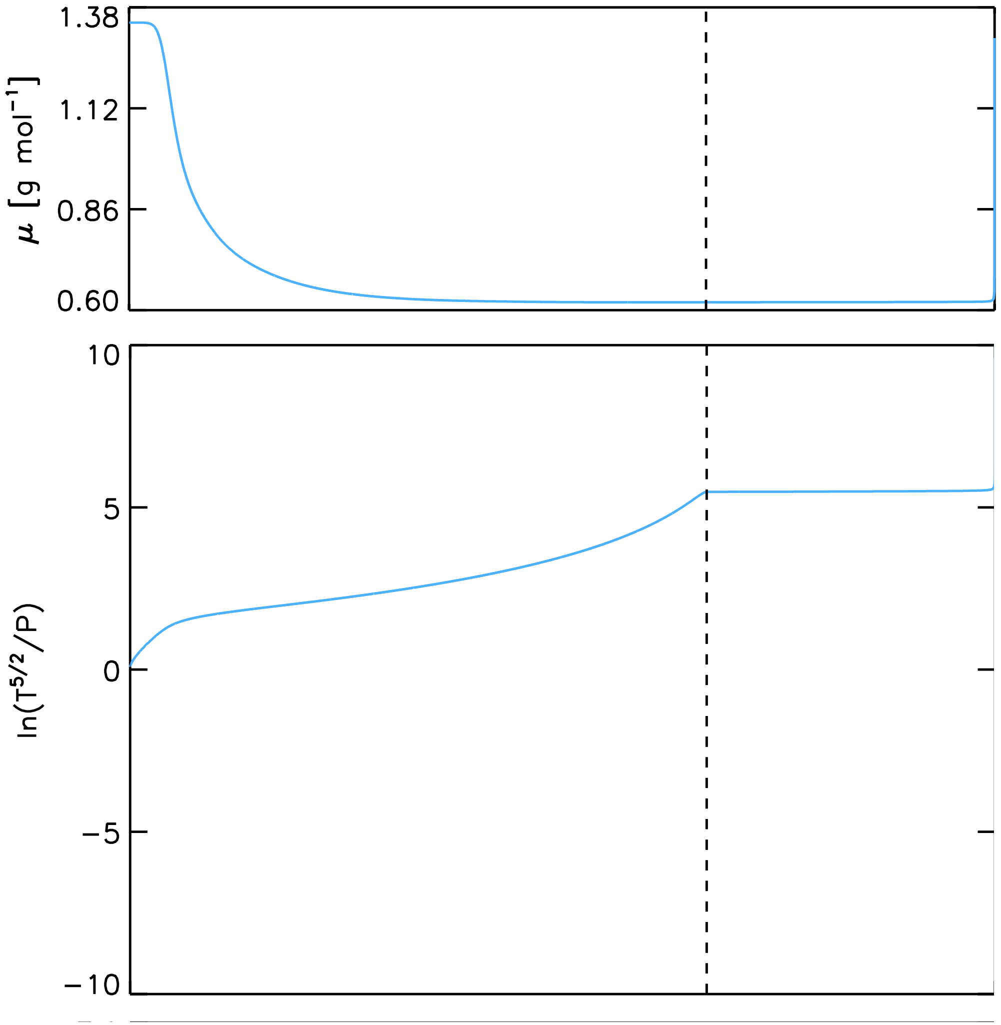
∇_{ad} = adiabatic temperature gradient with pressure
r = radius ordinate
c_P = specific heat at constant pressure

Specific entropy

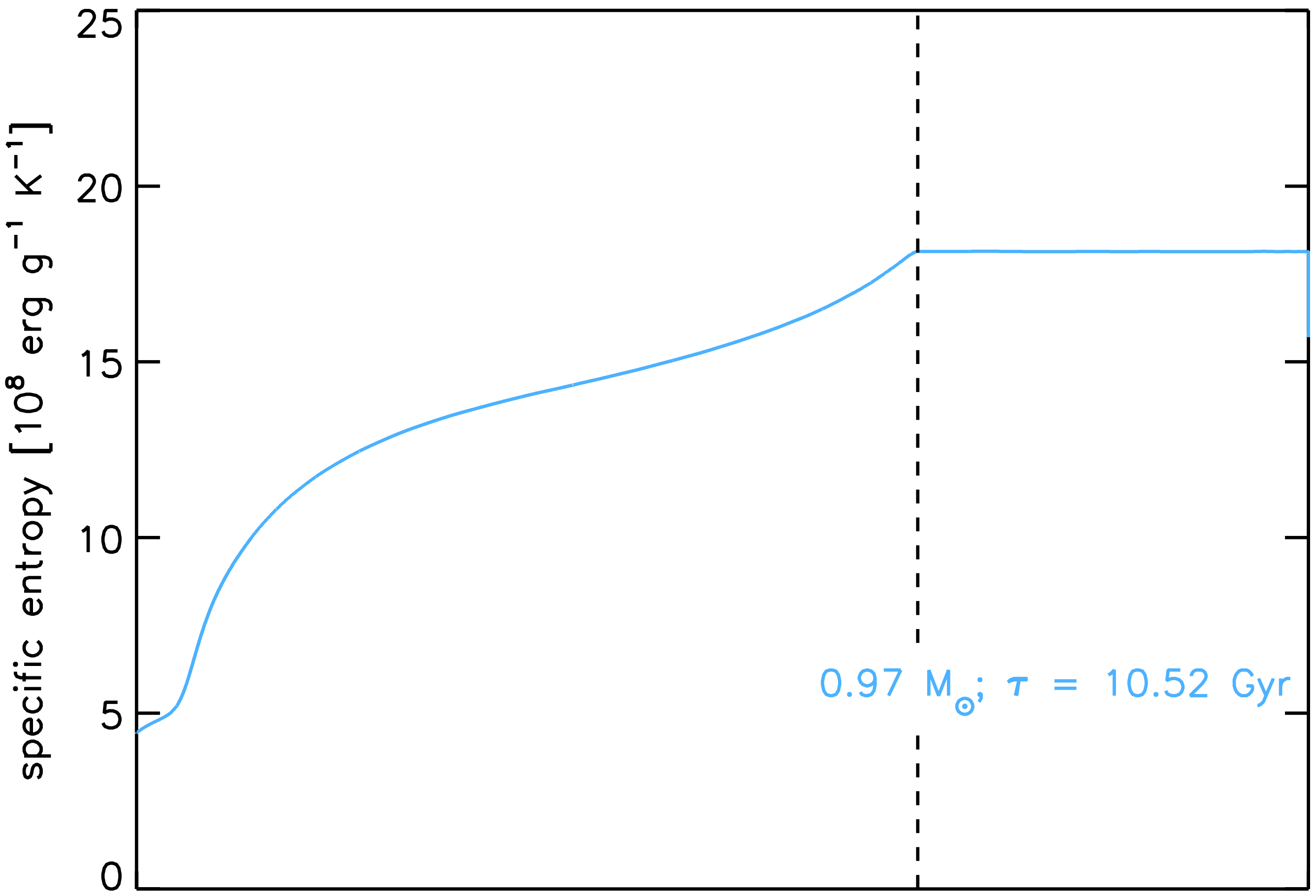


convective

Specific entropy: core



convective



convective

$$s = \frac{N_A k_B}{\mu} \ln\left[T^{5/2}/P\right]$$

Specific entropy with radius: radiative region

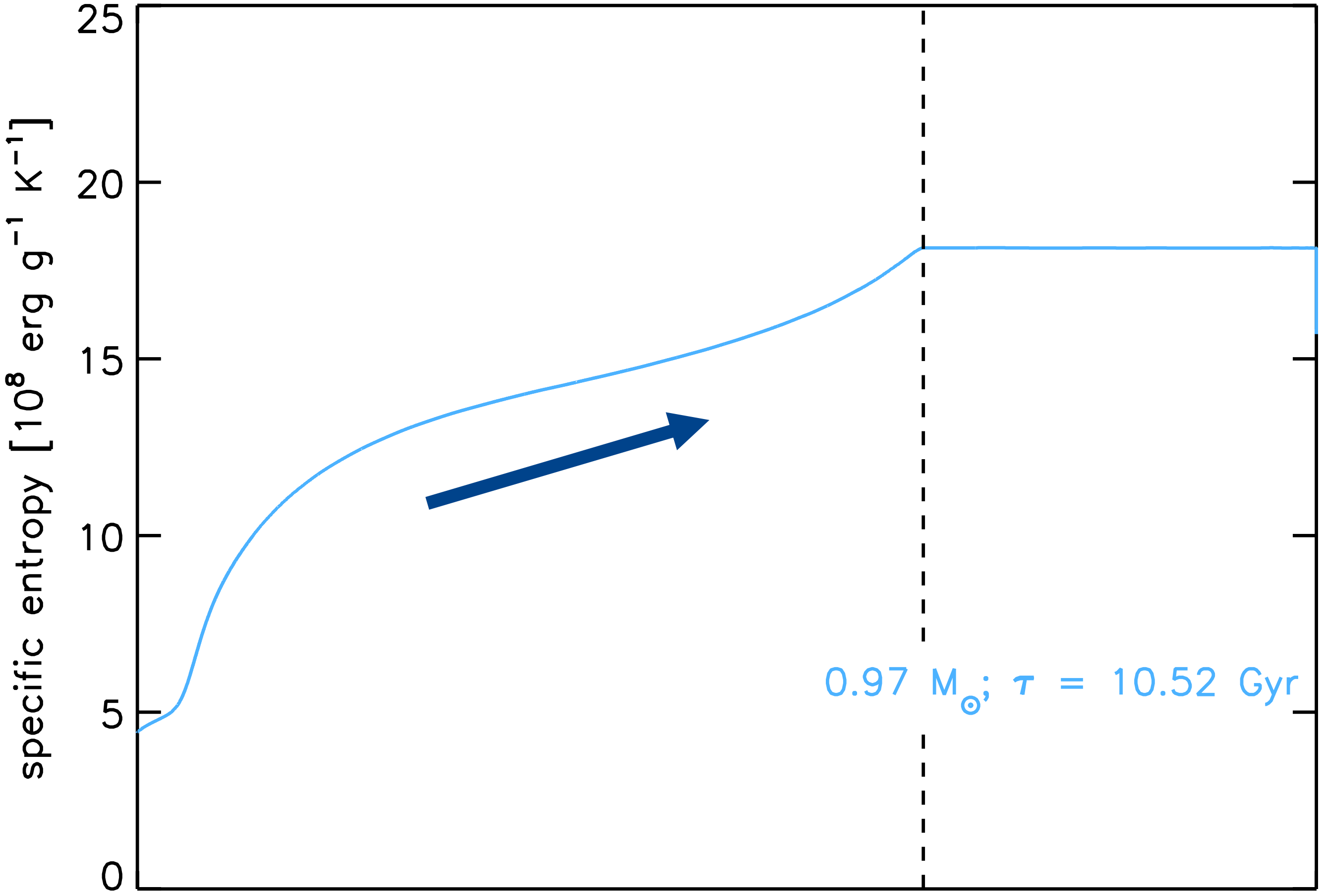
$$\frac{ds}{dr} = c_P(\nabla - \nabla_{ad})\frac{d\ln P}{dr}$$

hydrostatic equilibrium requires:

$$\frac{d\ln P}{dr} \leq 0 \qquad \nabla < \nabla_{ad}$$

$$\frac{ds}{dr} > 0$$

s increases outwards



convective

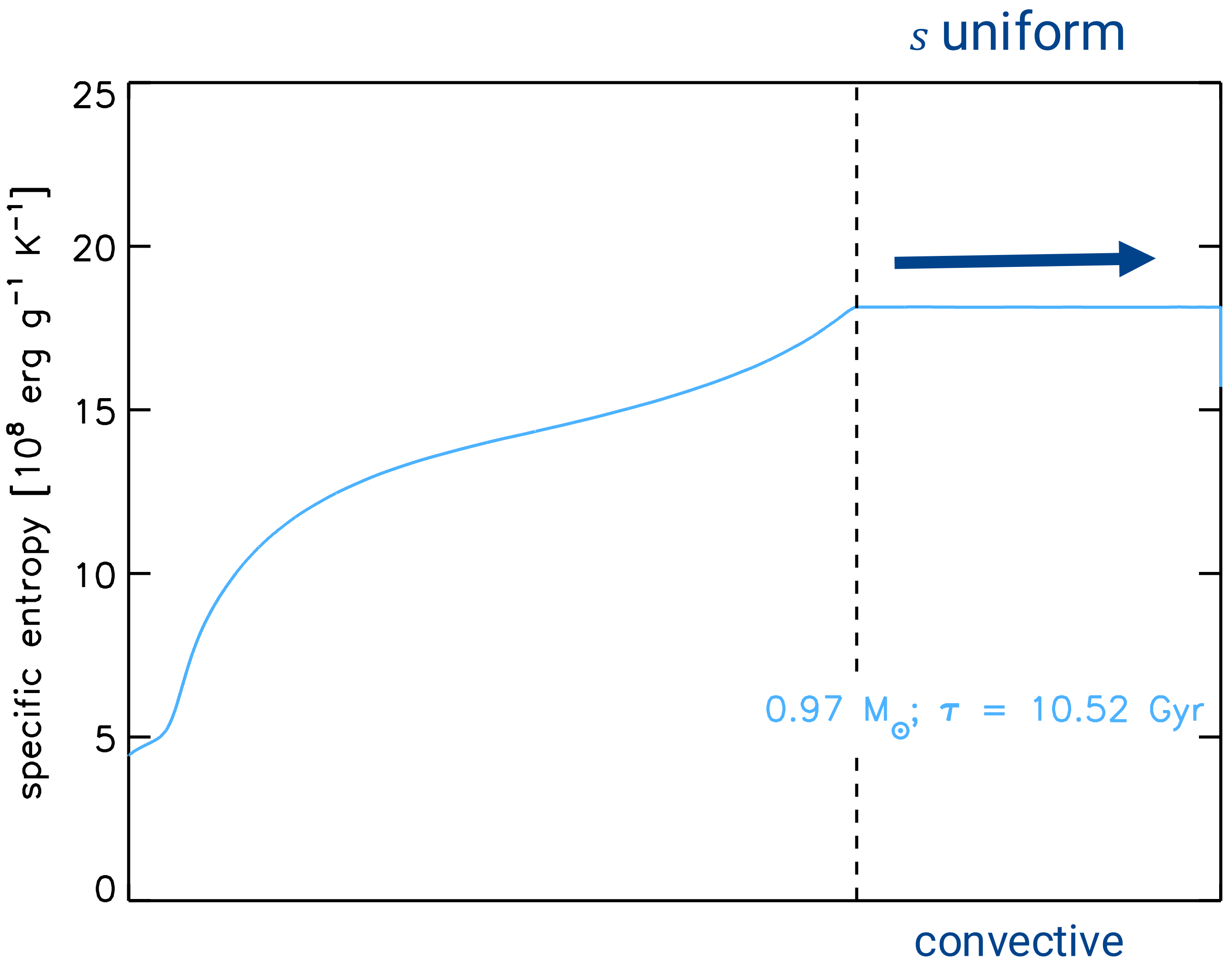
Specific entropy with radius: convective region

$$\frac{ds}{dr} = c_P(\nabla - \nabla_{ad})\frac{d\ln P}{dr}$$

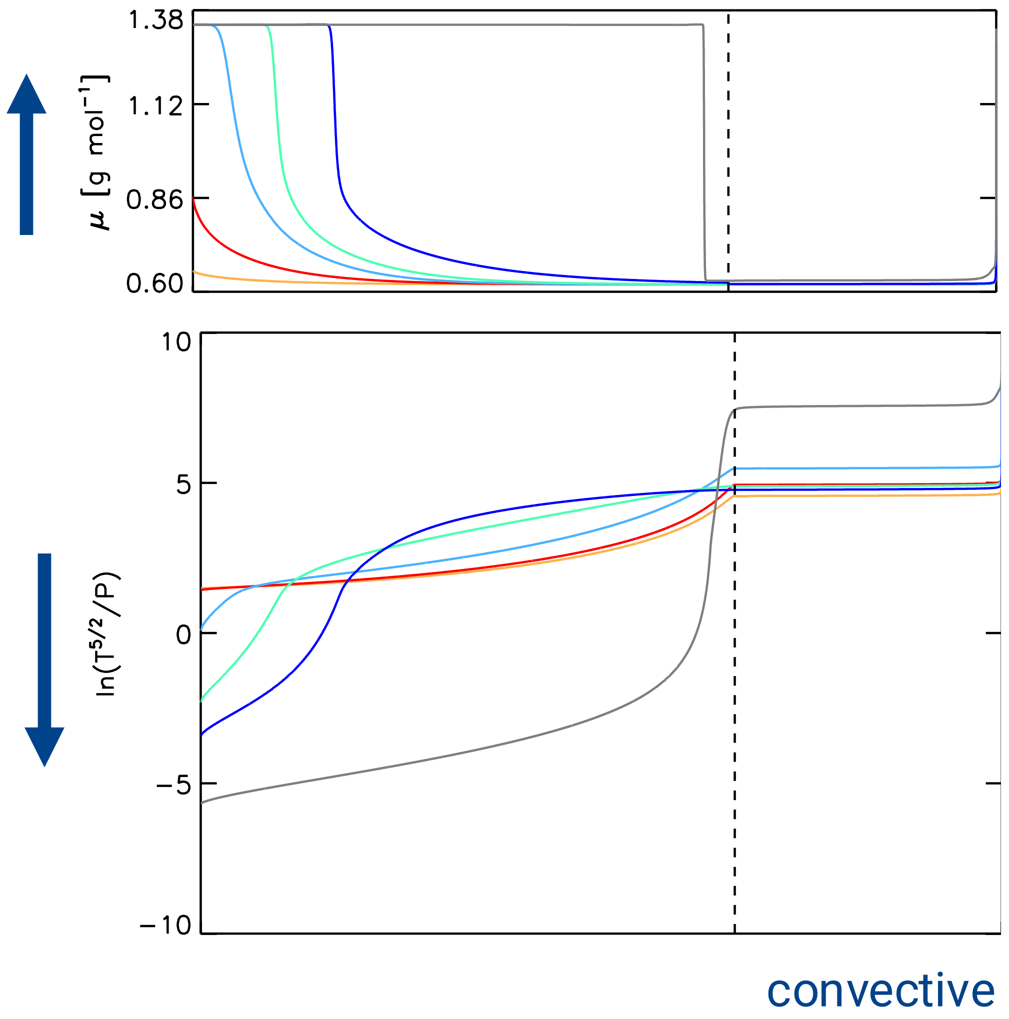
Convection is to good approximation isentropic:

$$\nabla = \nabla_{ad}$$

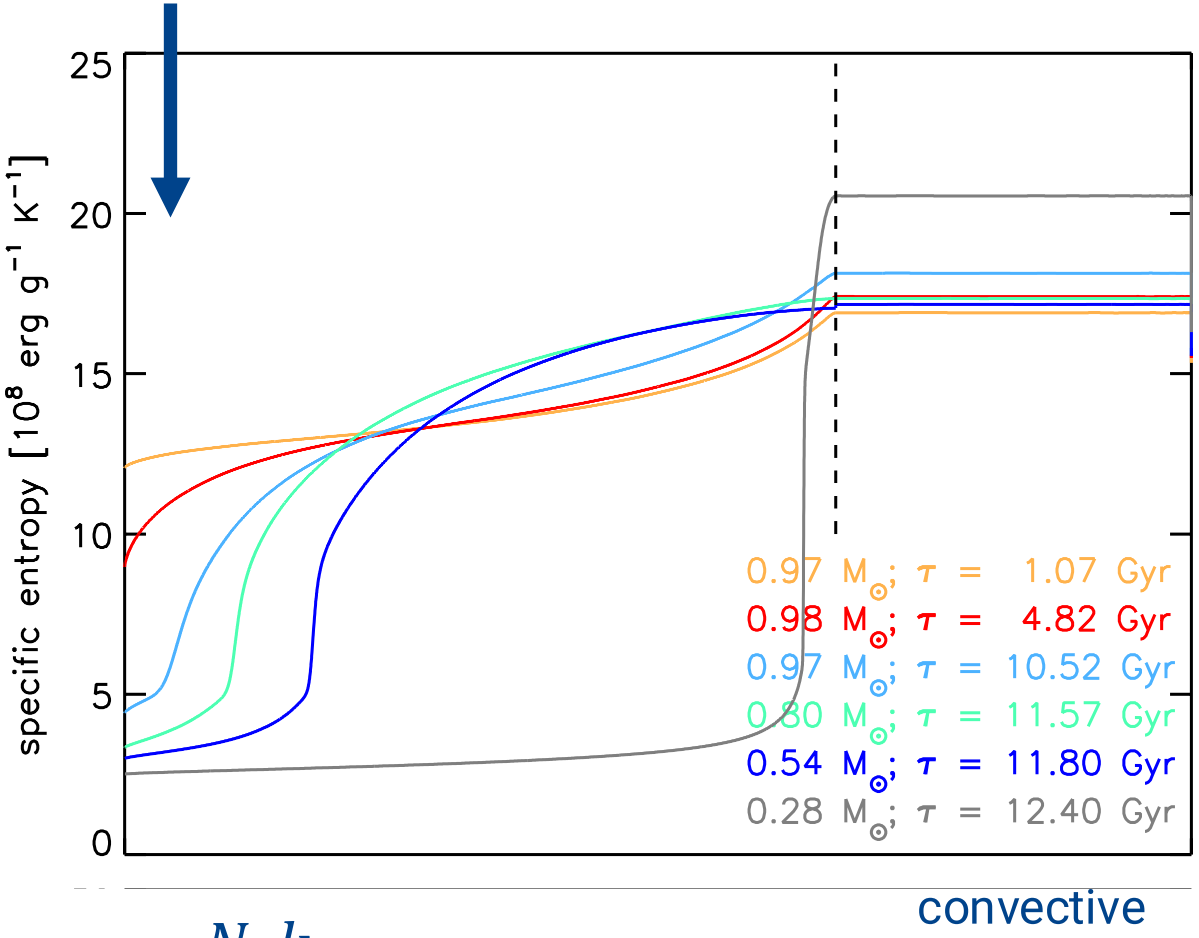
$$\frac{ds}{dr} = 0$$



Specific entropy with time: fusion region



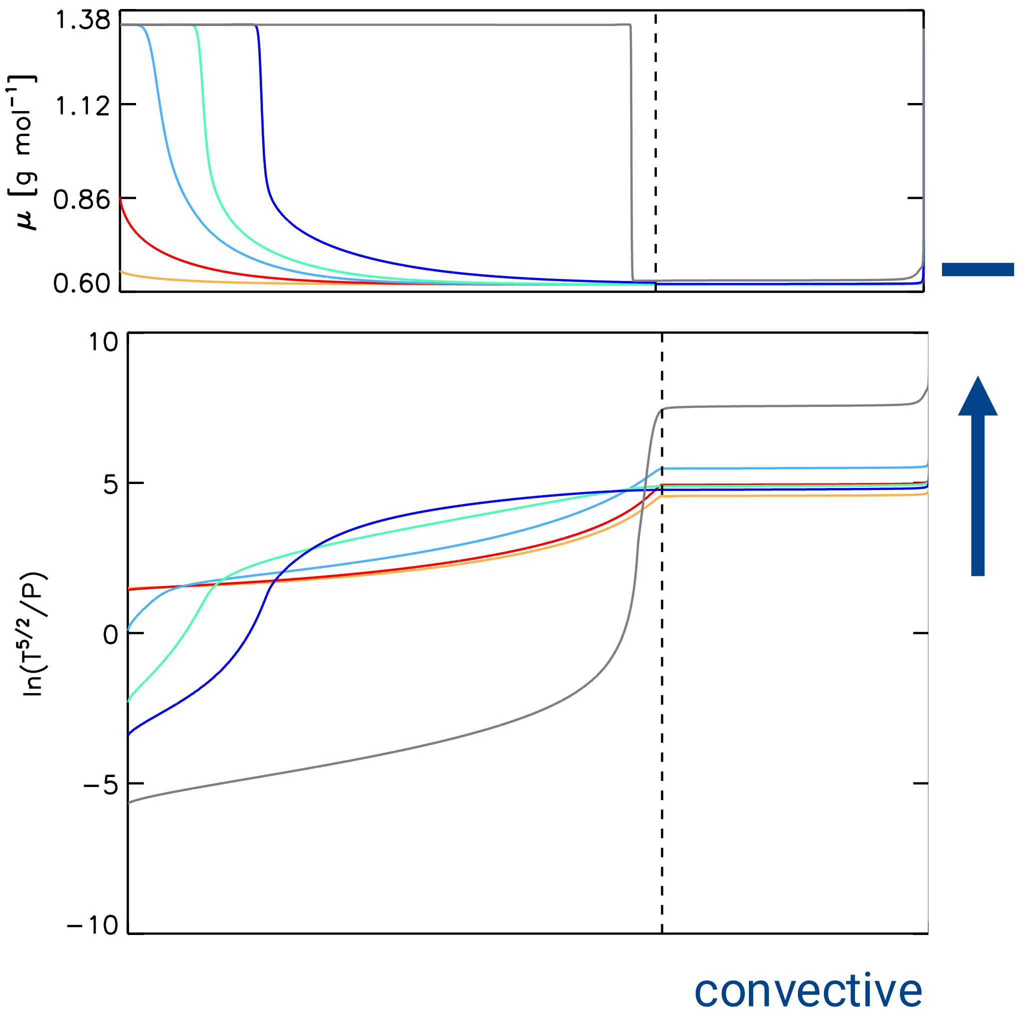
s decreases with time at the location where fusion is dominating

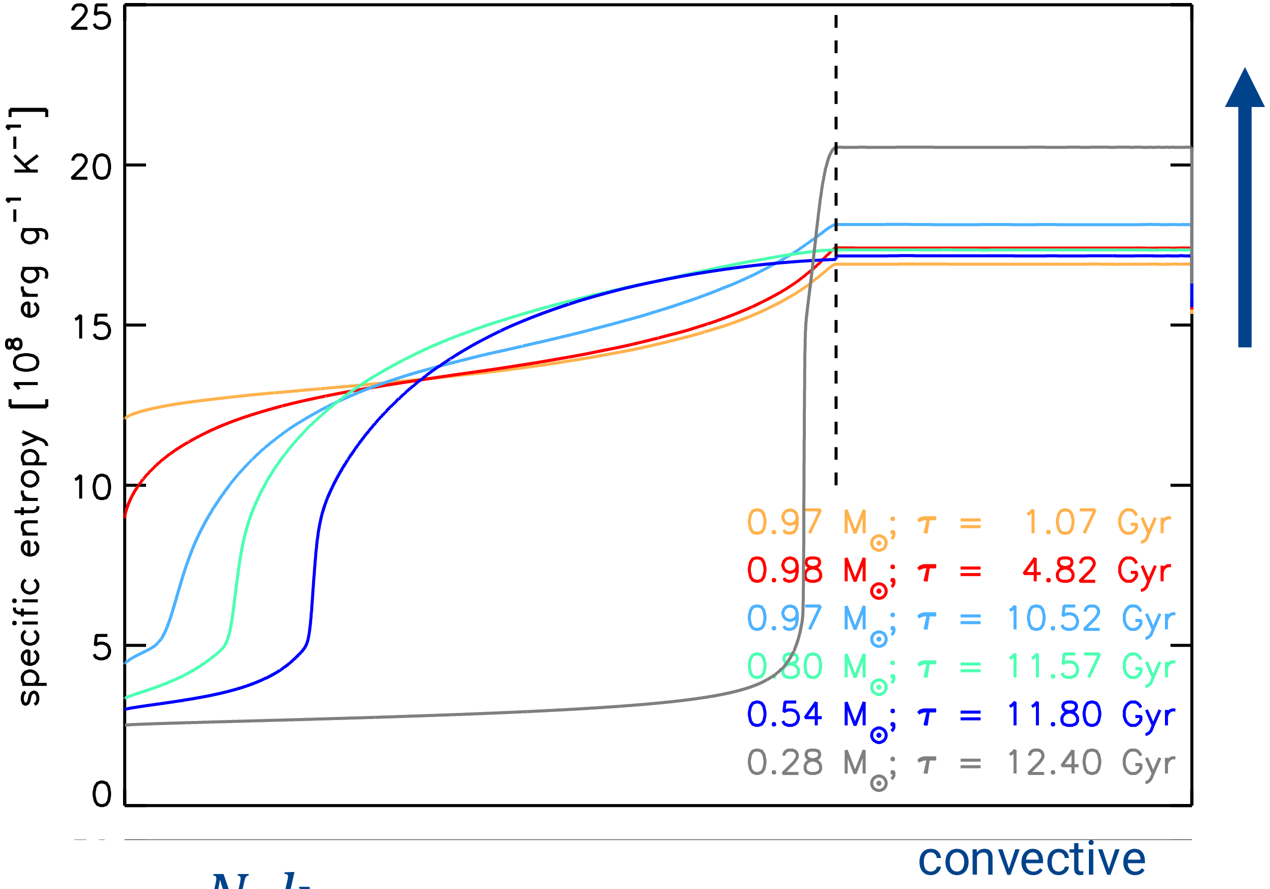


$$s = \frac{N_A k_B}{\mu} \ln\left[T^{5/2}/P\right]$$

Specific entropy with time: convective region

s increases with time in the convective region





$$s = \frac{N_A k_B}{\mu} \ln\left[T^{5/2}/P\right]$$

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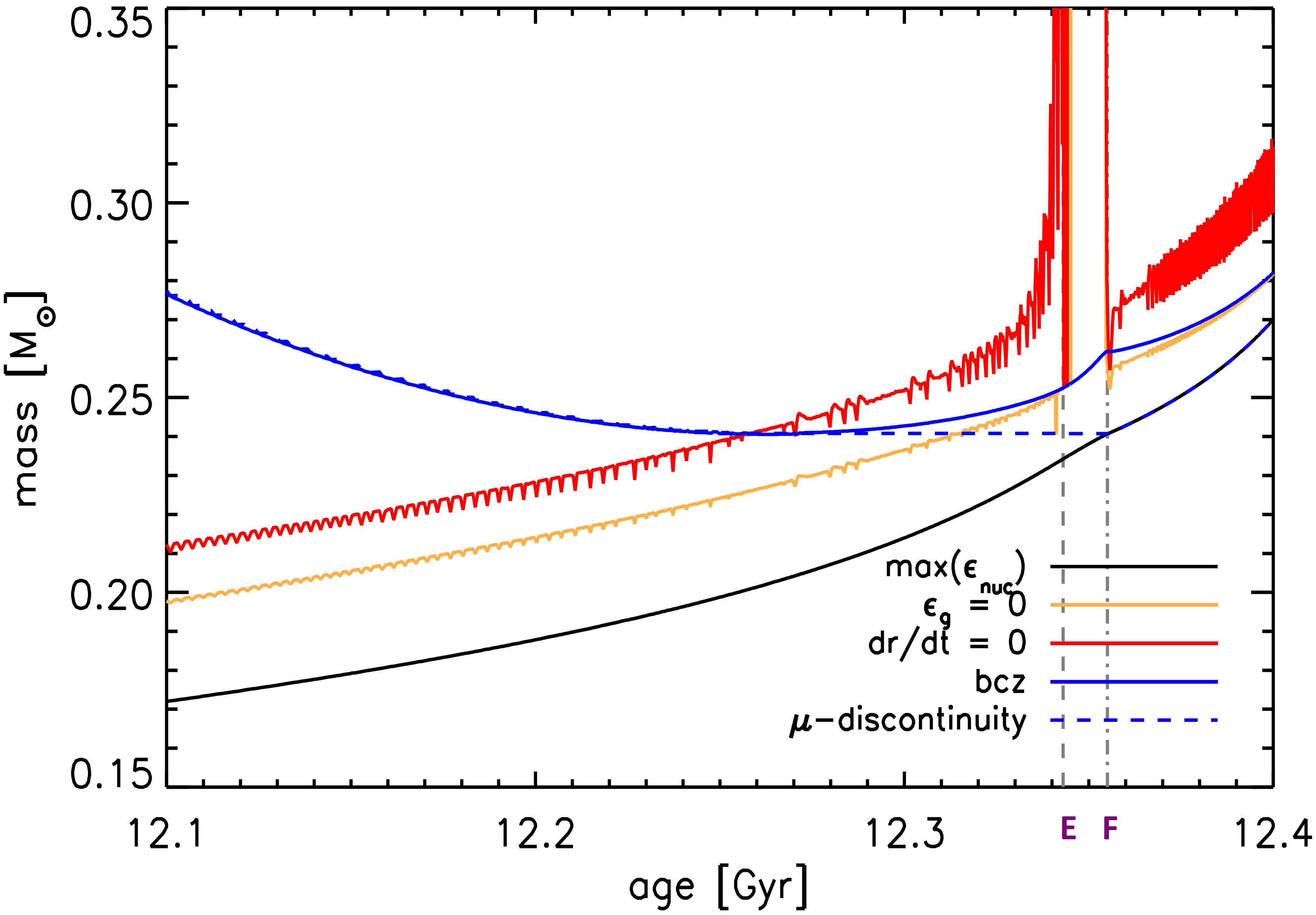
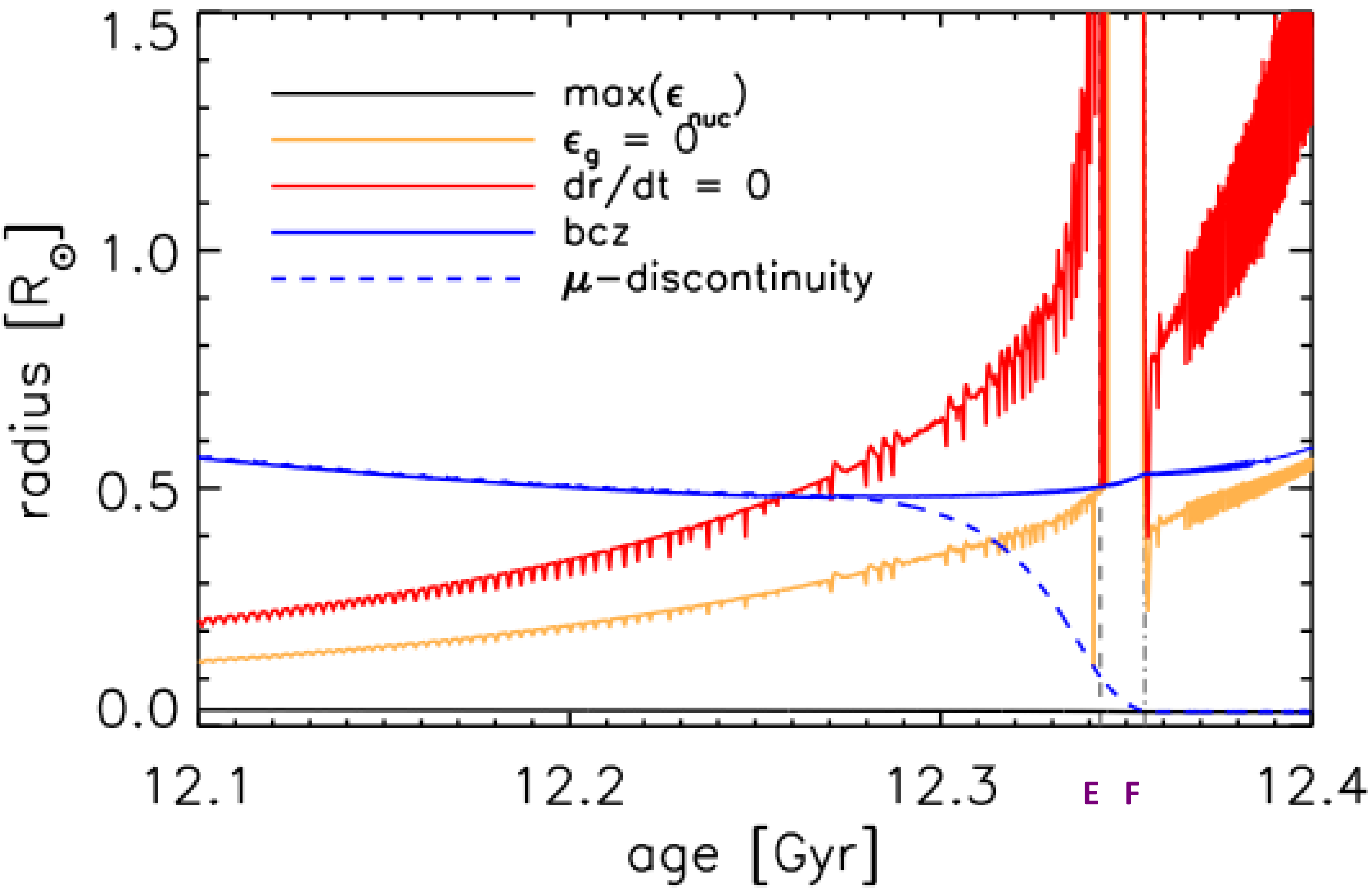
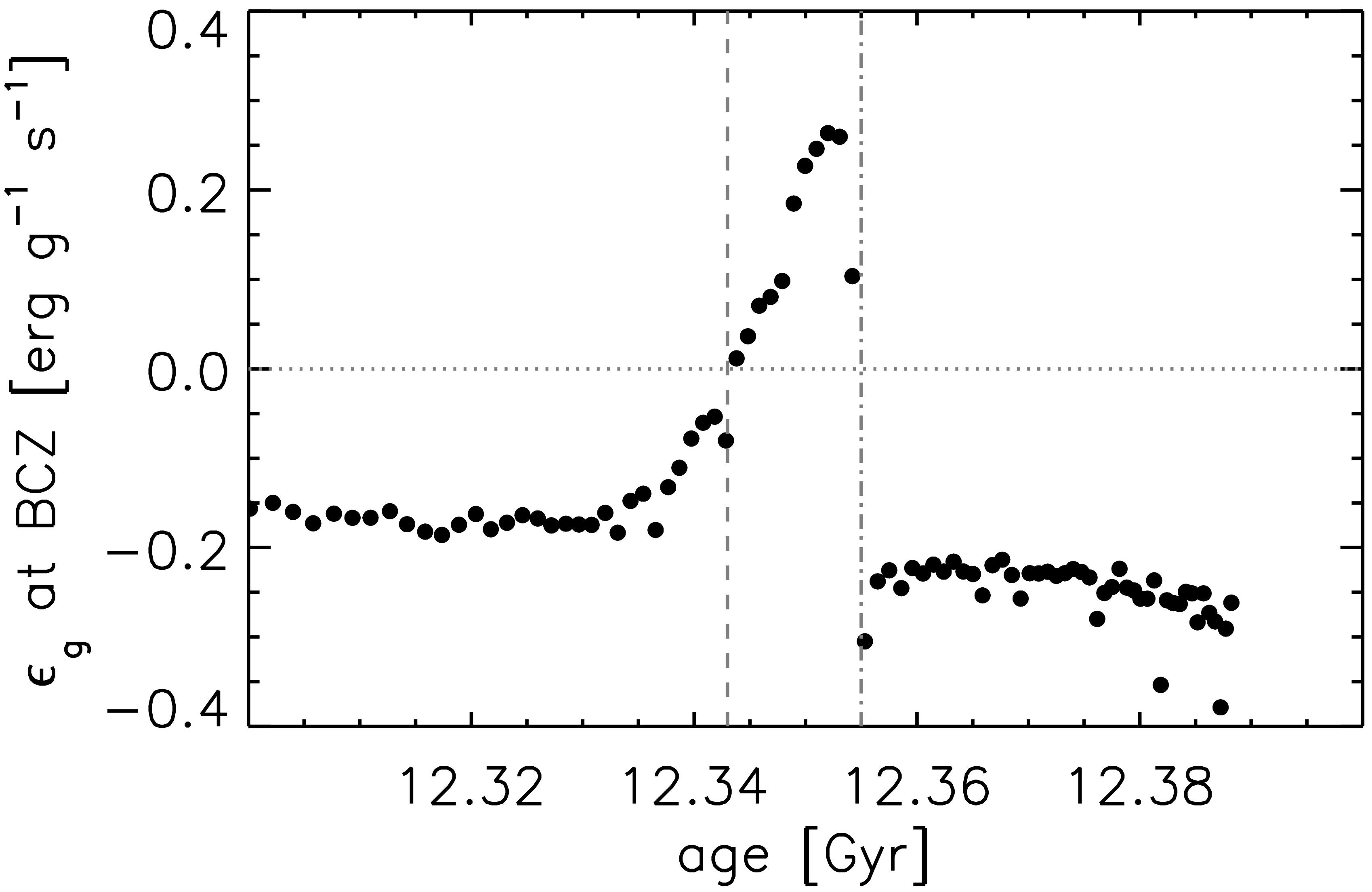
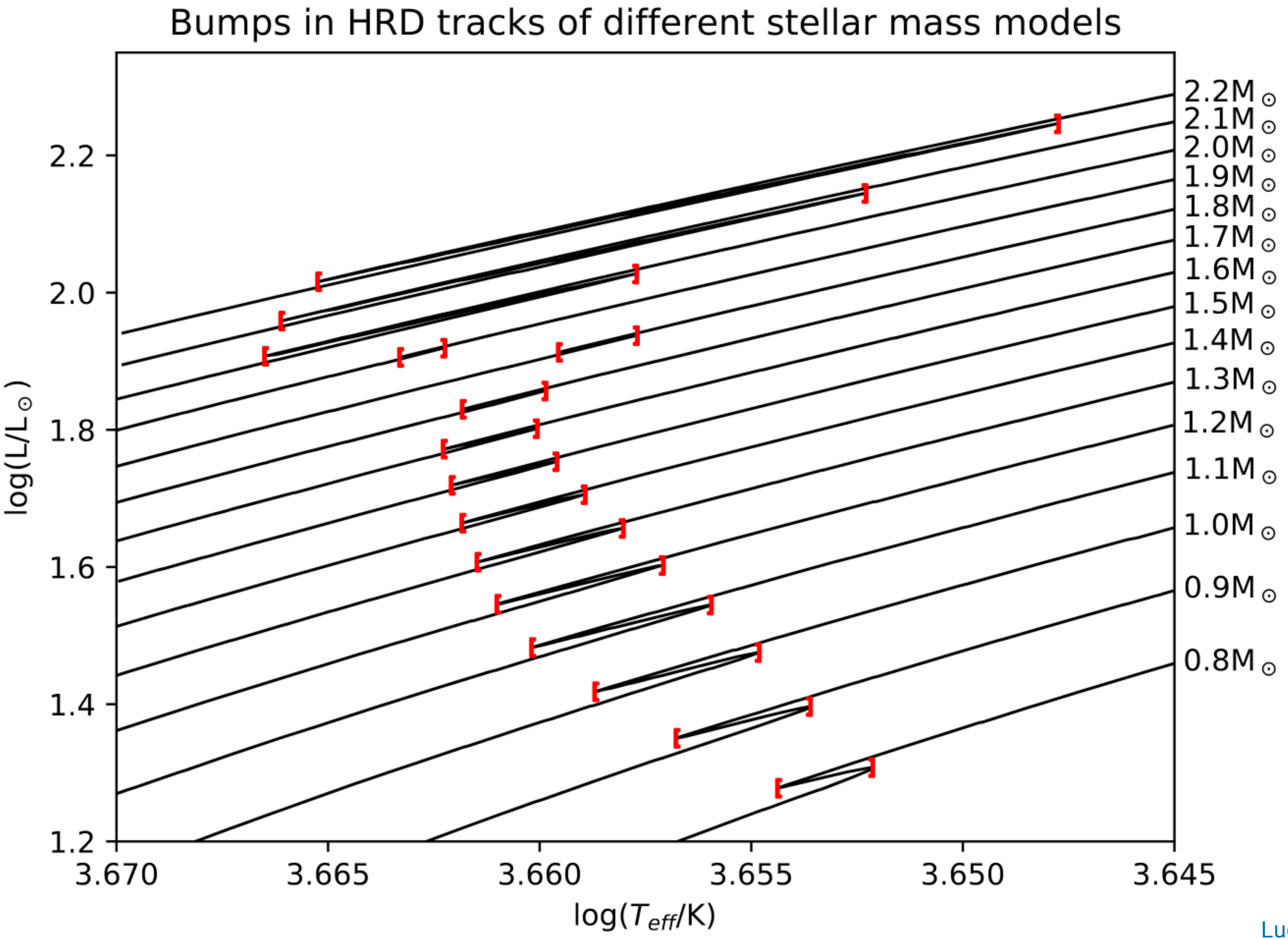


0.35
0.30
0.25
0.20
0.15
mass [M⊙]
12.1
12.2
12.3
E
F
12.4
age [Gyr]
max(ϵnuc)
ϵg = 0
dr/dt = 0
bcz
μ−discontinuity

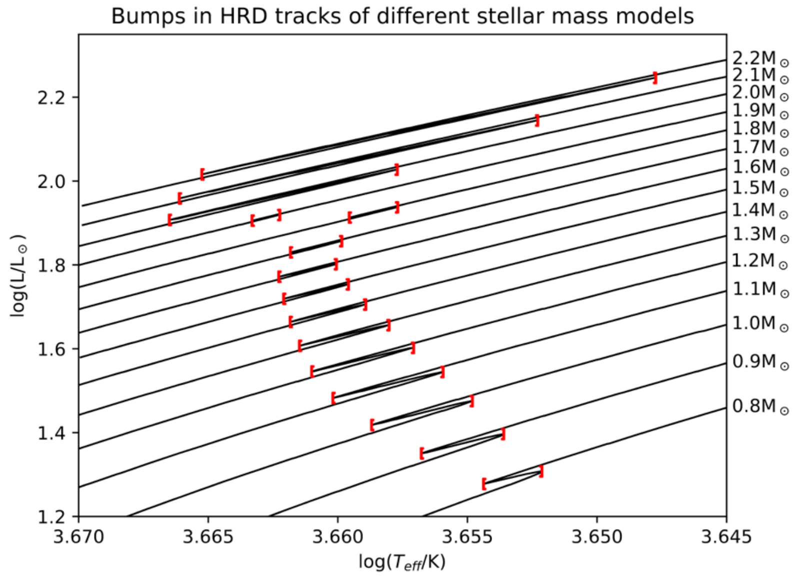

max(ϵ_{nuc})
ϵ_g = 0
dr/dt = 0
bcz
μ-discontinuity
radius [$R_\odot$]
age [Gyr]
1.5
1.0
0.5
0.0
12.1
12.2
12.3
12.4
E
F


0.4
0.2
0.0
−0.2
−0.4
ε_g at BCZ [erg g⁻¹ s⁻¹]
12.32
12.34
12.36
12.38
age [Gyr]

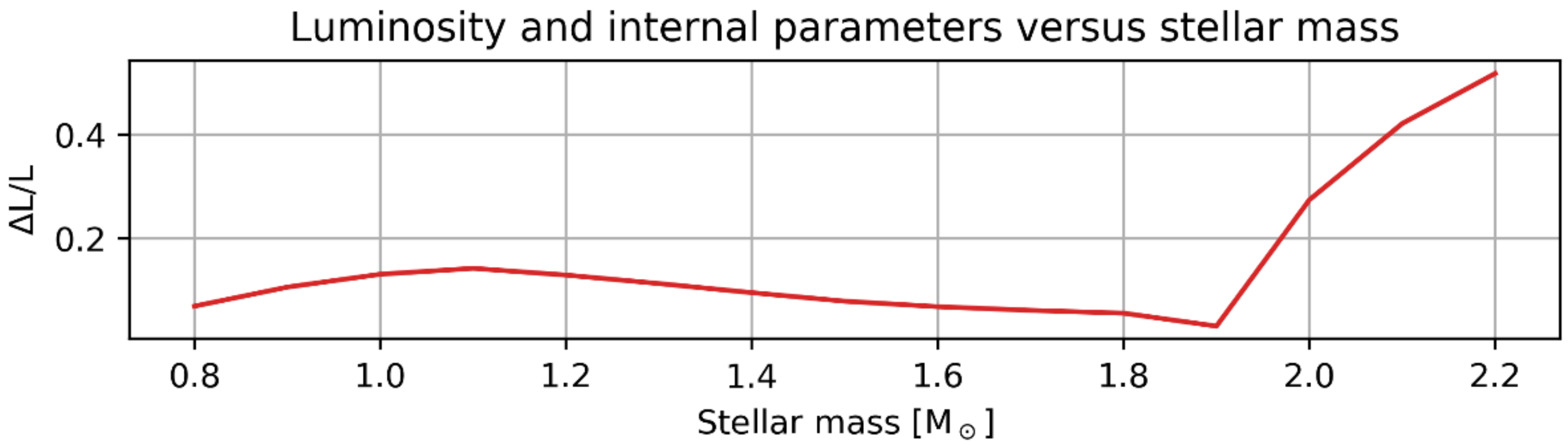
After Lab slides



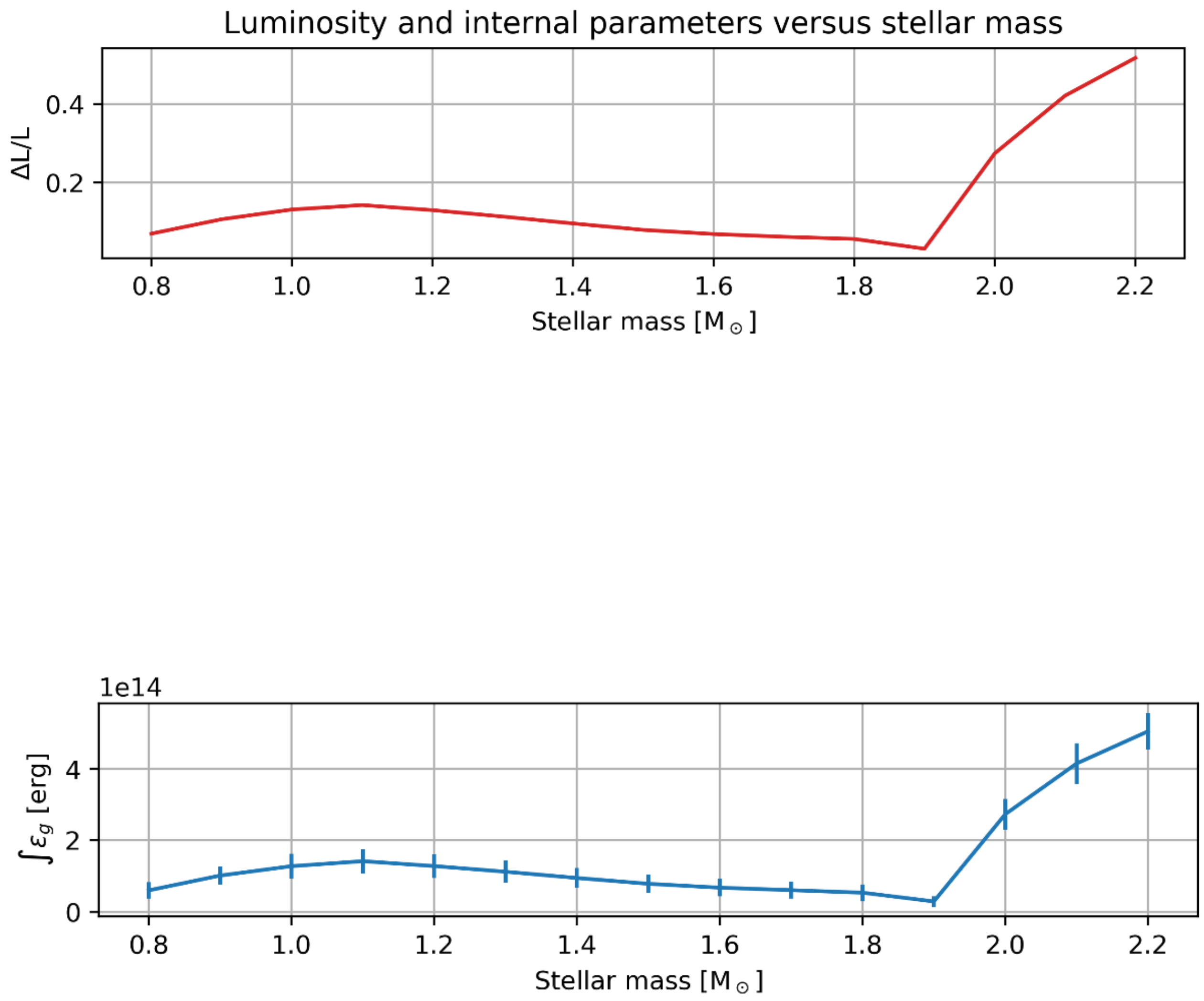
Bumps in HRD tracks of different stellar mass models
2.2M⊙
2.1M⊙
2.0M⊙
1.9M⊙
1.8M⊙
1.7M⊙
1.6M⊙
1.5M⊙
1.4M⊙
1.3M⊙
1.2M⊙
1.1M⊙
1.0M⊙
0.9M⊙
0.8M⊙
2.2
2.0
1.8
1.6
1.4
1.2
log(L/L⊙)
3.670
3.665
3.660
3.655
3.650
3.645
log(T_eff/K)

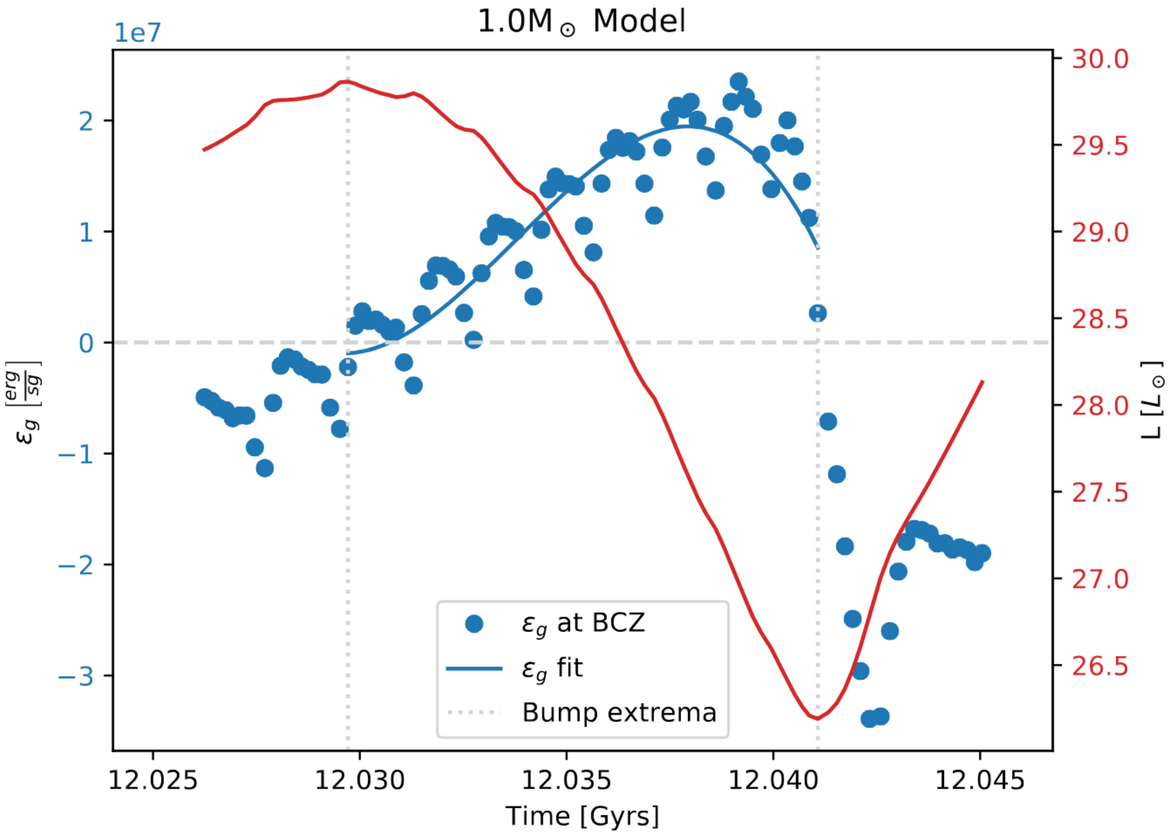


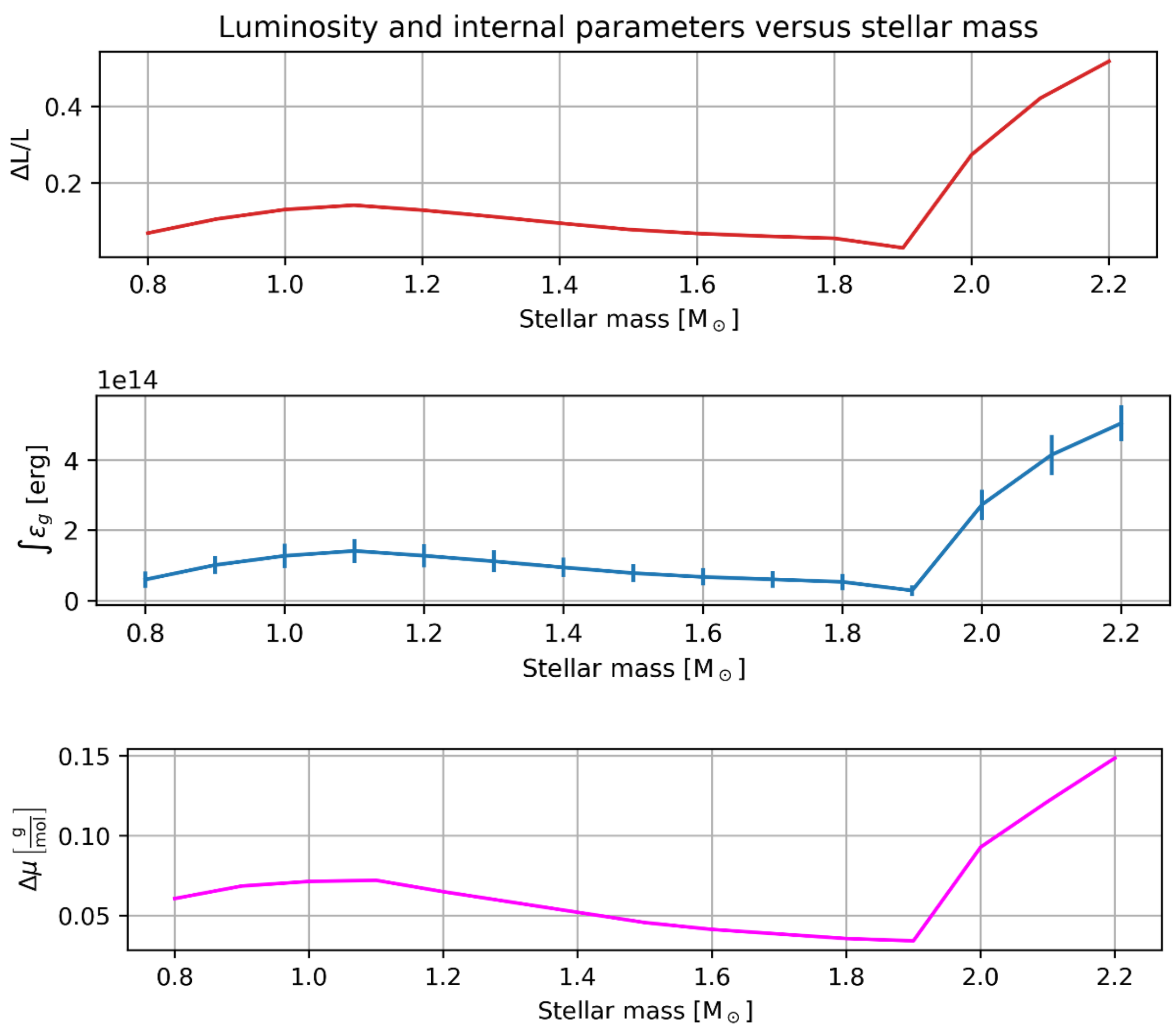
Bumps in HRD tracks of different stellar mass models
log(L/L⊙)
log(Teff/K)
2.2M⊙
2.1M⊙
2.0M⊙
1.9M⊙
1.8M⊙
1.7M⊙
1.6M⊙
1.5M⊙
1.4M⊙
1.3M⊙
1.2M⊙
1.1M⊙
1.0M⊙
0.9M⊙
0.8M⊙

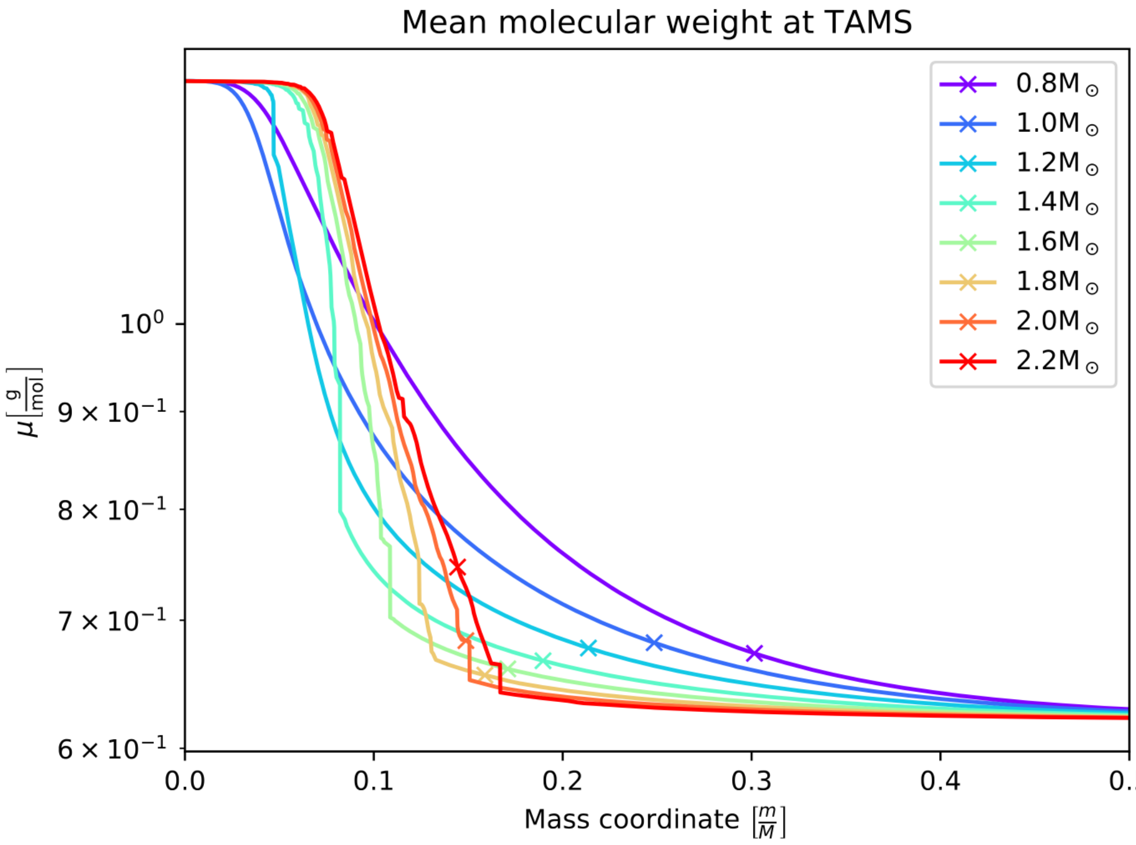


Luminosity and internal parameters versus stellar mass
ΔL/L
Stellar mass [M⊙]


Luminosity and internal parameters versus stellar mass
ΔL/L
0.4
0.2
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
Stellar mass [$M_\odot$]
1e14
$\int \epsilon_g$ [erg]
4
2
0
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
Stellar mass [$M_\odot$]


$1.0M_\odot$ Model
1e7
2
1
0
−1
−2
−3
ϵ_g $[\frac{erg}{sg}]$
30.0
29.5
29.0
28.5
28.0
27.5
27.0
26.5
L [$L_\odot$]
12.025
12.030
12.035
12.040
12.045
Time [Gyrs]
ϵ_g at BCZ
ϵ_g fit
Bump extrema


Luminosity and internal parameters versus stellar mass
ΔL/L
0.4
0.2
Stellar mass [M⊙]
1e14
∫εg [erg]
4
2
0
Stellar mass [M⊙]
Δμ [g/mol]
0.15
0.10
0.05
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
Stellar mass [M⊙]


Mean molecular weight at TAMS
0.8M⊙
1.0M⊙
1.2M⊙
1.4M⊙
1.6M⊙
1.8M⊙
2.0M⊙
2.2M⊙
μ [g/mol]
10^0
9×10^{-1}
8×10^{-1}
7×10^{-1}
6×10^{-1}
0.0
0.1
0.2
0.3
0.4
0.5
Mass coordinate [m/M]